



# RESEARCH MEMORANDUM

ANALYSIS OF HEAT TRANSFER AND FLUID FRICTION FOR FULLY  
DEVELOPED TURBULENT FLOW OF SUPERCRITICAL WATER  
WITH VARIABLE FLUID PROPERTIES IN A SMOOTH TUBE

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This is a  
PROPOSED REPORT  
NO. YET APPROVED  
FOR PUBLICATION

(NACA-RM-E53B17) ANALYSIS OF HEAT  
TRANSFER AND FLUID FRICTION FOR FULLY  
DEVELOPED TURBULENT FLOW OF SUPERCRITICAL  
WATER WITH VARIABLE FLUID PROPERTIES IN  
A SMOOTH TUBE (NASA) 32 p

N73-74683

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ANALYSIS OF HEAT TRANSFER AND FLUID FRICTION FOR FULLY  
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SUMMARY

A previous analysis of turbulent flow and heat transfer for air with variable properties flowing in smooth tubes is generalized in order to make it applicable to supercritical water. The generalization is necessary because all the pertinent properties of supercritical water vary markedly with temperature. Calculated velocity and temperature distributions, as well as relations among Nusselt number, Reynolds number, and friction factor, are presented. The effect of variation of fluid properties across the tube on the Nusselt number and friction factor correlations can be eliminated by evaluating the properties at a reference temperature which is a function of both the wall temperature and the ratio of wall-to-bulk temperatures.

INTRODUCTION

An extension and generalization of the analysis for air with variable properties given in reference 1 were made in order to make it applicable to supercritical water. (The term "supercritical water" designates water at pressures above the critical pressure and temperatures near the critical temperature.) An important characteristic of supercritical water, insofar as it affects heat transfer and flow, is the very large variation with temperature of its physical properties (specific heat, density, viscosity, and thermal conductivity). Plots of the variation of the property values of water with temperature at a pressure of 5000 pounds per square inch is given in figures 1 and 2 for data from references 2 to 4. From these plots it is evident that the radial variation of properties for supercritical water flowing in a tube with heat transfer should have a considerable effect on the velocity and temperature distributions, with a resultant effect on the heat-transfer coefficients and friction factors.

Several other analyses of heat transfer to supercritical water, which used simplifying assumptions not made herein, have been made and are reported in reference 3.

### ANALYSIS

When the velocities and temperatures in a smooth tube are to be obtained as functions of distance from the wall, the differential equations for shear stress and heat transfer may be written as follows:

$$\tau = \mu \frac{du}{dy} + \rho \epsilon \frac{du}{dy} \quad (1)$$

$$q = -k \frac{dt}{dy} - \rho g c_p \epsilon_h \frac{dt}{dy} \quad (2)$$

Equations (1) and (2) written in dimensionless form become

$$\frac{\tau}{\tau_0} = \frac{\mu}{\mu_0} \frac{du^+}{dy^+} + \frac{\rho}{\rho_0} \frac{\epsilon}{\mu_0} \frac{du^+}{dy^+} \quad (3)$$

$$\frac{q}{q_0} = \left( \frac{k}{k_0} \frac{1}{Pr_0} + \frac{\rho}{\rho_0} \frac{c_p}{c_{p,0}} \frac{\epsilon_h}{\mu_0} \right) \frac{dt^+}{dy^+} \quad (4)$$

(The symbols used in this report are defined in the appendix.)

Assumptions. - The following assumptions are made in the use of equations (1) and (2) for obtaining velocity and temperature distributions with heat transfer:

(1) The eddy diffusivities for momentum and heat transfer  $\epsilon$  and  $\epsilon_h$  are equal. Previous analyses for flow in tubes based on this assumption yielded heat-transfer coefficients and friction factors that agree with experiment except at low Peclet numbers. At low Peclet numbers the ratio of eddy diffusivities appears to be a function of Peclet number ( $Pe = Re Pr$ ), but for Peclet numbers above 10,000, which is the range of interest for this analysis, the diffusivities are nearly equal (ref. 5).

(2) The expressions for eddy diffusivity that are found to apply to flow of air with heat transfer with variable properties (ref. 1) apply also to supercritical water. These expressions are

$$\epsilon = n^2 u y \quad (5)$$

for flow close to the wall, and the Karman relation

$$\epsilon = \kappa^2 \frac{\left(\frac{du}{dy}\right)^3}{\left(\frac{d^2u}{dy^2}\right)^2} \quad (6)$$

for flow far from the wall, where  $\kappa$  and  $\kappa$  are experimental constants having the values 0.109 and 0.36, respectively. In the expression for flow close to the wall, the effect of kinematic viscosity  $\mu/\rho$  on  $\epsilon$  has been neglected. A preliminary investigation (not reported herein) indicates that neglect of this effect of  $\mu/\rho$  causes a somewhat larger variation of Nusselt number with Prandtl number than is indicated experimentally.

(3) The variations of the shear stress  $\tau$  and the heat transfer per unit area  $q$  across the tube have a negligible effect on the velocity and temperature distributions.

Flow close to wall. - For obtaining the velocity and temperature distributions close to a smooth wall, the expression for  $\epsilon$  from equation (5) is substituted into equations (3) and (4) to give, in terms of the dimensionless parameters  $u^+$  and  $y^+$ ,

$$1 = \left( \frac{\mu}{\mu_0} + \frac{\rho}{\rho_0} n^2 u^+ y^+ \right) \frac{du^+}{dy^+} \quad (7)$$

and

$$1 = \left( \frac{k}{k_0} \frac{1}{Pr_0} + \frac{\rho}{\rho_0} \frac{c_p}{c_{p,0}} n^2 u^+ y^+ \right) \frac{dt^+}{dy^+} \quad (8)$$

where assumptions (1), (2), and (3) have been used.

By rearrangement, equations (7) and (8) become

$$du^+ = \frac{dy^+}{\frac{\mu}{\mu_0} + \frac{\rho}{\rho_0} n^2 u^+ y^+} \quad (9)$$

$$dt^+ = \frac{dy^+}{\frac{k}{k_0} \frac{1}{Pr_0} + \frac{\rho}{\rho_0} \frac{c_p}{c_{p,0}} n^2 u^+ y^+} \quad (10)$$

Equations (9) and (10) can then be written in integral form as

$$u^+ = \int_0^{y^+} \frac{dy^+}{\frac{\mu}{\mu_0} + \frac{\rho}{\rho_0} n^2 u^+ y^+} \quad (11)$$

and

$$t^+ = \int_0^{y^+} \frac{dy^+}{\frac{k}{k_0} \frac{1}{Pr_0} + \frac{\rho}{\rho_0} \frac{c_p}{c_{p,0}} n^2 u^+ y^+} \quad (12)$$

For a given wall temperature,  $k/k_0$ ,  $\mu/\mu_0$ ,  $\rho/\rho_0$ , and  $c_p/c_{p,0}$  are functions of  $t/t_0$ ; and since  $t/t_0 = 1 - \beta t^+$  ( $\beta = \frac{q_0 \sqrt{\tau_0/\rho_0}}{c_{p,0} g \tau_0 t_0}$ ),  $k/k_0$ ,  $\mu/\mu_0$ ,  $\rho/\rho_0$ , and  $c_p/c_{p,0}$  are functions of  $\beta t^+$ . For given values of  $t_0$  and  $\beta$ , equations (11) and (12) can be solved simultaneously by iteration, that is, assumed relations between  $u^+$  and  $y^+$  and between  $t^+$  and  $y^+$  are substituted into the right-hand sides of the equations and new values of  $u^+$  and  $t^+$  are calculated by numerical integration. These new values are then substituted into the right-hand sides of the equations and the process is repeated until the values of  $u^+$  and  $t^+$  do not change appreciably. Equations (11) and (12) give the relations among  $u^+$ ,  $t^+$ , and  $y^+$  for various values of the heat-transfer parameter  $\beta$  for flow close to the wall.

Flow at a distance from wall. - By use of assumptions (1), (2), and (3), equations (3) and (4) become, for flow at a distance from the wall,

$$1 = \frac{\mu}{\mu_0} \frac{du^+}{dy^+} + \frac{\rho}{\rho_0} x^2 \frac{\left(\frac{du^+}{dy^+}\right)^4}{\left(\frac{d^2 u^+}{dy^+ 2}\right)^2} \quad (13)$$

$$1 = \left[ \frac{k}{k_0} + \frac{\rho}{\rho_0} \frac{c_p}{c_{p,0}} x^2 \frac{\left(\frac{du^+}{dy^+}\right)^3}{\left(\frac{d^2 u^+}{dy^+ 2}\right)^2} \right] \frac{dt^+}{dy^+} \quad (14)$$

In equations (13) and (14), the molecular-shear-stress and heat-transfer terms have been retained so that the slopes for the velocity and temperature distributions close to and far from the wall can be matched at  $y_1^+$ . From the analysis for air presented in reference 1, it was found that  $y_1^+$ , which is the lowest value of  $y^+$  for which the equations for flow at a distance from the wall apply, has a value of 16 when the molecular-shear-stress and heat-transfer terms are retained. Substitution of  $v$  for  $du^+/dy^+$  in equation (13) and rearrangement of the terms give

$$\frac{dv}{v} = -x \frac{du^+}{\sqrt{\frac{1 - (\mu/\mu_0)v}{\rho/\rho_0}}} \quad (15)$$

which written in integral form is

$$v = v_1 e^{-x} \int_{u_1^+}^{u^+} \frac{du^+}{\sqrt{\frac{1 - (\mu/\mu_0)v}{\rho/\rho_0}}} \quad (16)$$

Substitution of  $v$  for  $du^+/dy^+$  in equation (14) and rearrangement of the terms give

$$\frac{dv}{v} = -x \sqrt{\frac{(\rho/\rho_0)(c_p/c_{p,0})}{\frac{v}{dt^+/dy^+} - \frac{v k/k_0}{Pr_0}}} du^+ \quad (17)$$

Elimination of  $dv/v$  from equations (15) and (17), rearrangement, and integration result in

$$t^+ = t_1^+ + \int_{u_1^+}^{u^+} \frac{du^+}{\frac{c_p}{c_{p,0}} + v \left( \frac{k/k_0}{Pr_0} - \frac{c_p}{c_{p,0}} \frac{\mu}{\mu_0} \right)} \quad (18)$$

If relations among  $v$ ,  $u^+$ , and  $t^+$  are assumed, equations (16) and (18) can be solved simultaneously by iteration. After the relation among  $v$ ,  $t^+$ , and  $u^+$  is determined, the expression  $v = \frac{du^+}{dy^+}$  or

$$y^+ = y_1^+ + \int_{u_1^+}^{u^+} \frac{du^+}{v} \quad (19)$$

can be integrated, and  $y^+$  can be calculated.

Nusselt number, Reynolds number, and friction factor. - By use of the definitions of the quantities involved, it can be shown (see ref. 6) that with the fluid properties evaluated at the wall temperature, the Nusselt number, Reynolds number, and friction factor are given by

$$Nu_0 = \frac{2r_0^+ \Pr}{t_b^+} \quad (20)$$

$$Re_0 = 2r_0^+ u_b^+ \quad (21)$$

$$f_0 = \frac{2}{u_b^+ + 2} \quad (22)$$

where, for variable  $c_p$  and  $\rho$ ,

$$t_b^+ = \frac{\int_0^{r_0^+} \frac{c_p}{c_{p,0}} \frac{\rho}{\rho_0} t^+ u^+ (r_0^+ - y^+) dy^+}{\int_0^{r_0^+} \frac{c_p}{c_{p,0}} \frac{\rho}{\rho_0} u^+ (r_0^+ - y^+) dy^+}$$

and

$$u_b^+ = \frac{2}{r_0^+ + 2} \int_0^{r_0^+} u^+ (r_0^+ - y^+) dy^+$$

The relations between  $u^+$ ,  $y^+$ , and  $t^+$  are calculated from equations (11), (12), (16), (18), and (19).

The Nusselt number, Reynolds number, and friction factor with the fluid properties evaluated at the bulk temperature rather than at the wall temperature can be found by using the relation  $t_b/t_0 = 1 - \beta t_b^+$ .

Nusselt numbers, Reynolds numbers, and friction factors with properties evaluated at an arbitrary temperature in the fluid  $t_x$  can be obtained from  $t_x/t_0 = x(1 - t_b/t_0) + t_b/t_0$ .

## RESULTS AND DISCUSSION

### Physical Properties

The physical properties used in this analysis are shown in figures 1 and 2. All the properties used were taken from tables and equations given in reference 2, with the exception of the thermal conductivities and viscosities at the lower temperatures which were taken from reference 4. Considerable uncertainty exists in the property values of supercritical water, especially for viscosity and thermal conductivity, so that the accuracy of the computed results may be affected to some extent by errors in those values. The discrepancies among the viscosity and conductivity data used by various investigators are shown by the difference between the solid and dashed lines in figure 2.

### Predicted Temperature and Velocity Distribution

The predicted velocity and temperature distributions for various values of  $\beta$  for a wall temperature of  $1360^\circ$  R and a pressure of 5000 pounds per square inch are shown in figures 3 and 4, respectively. Since the molecular-shear-stress and heat-transfer terms were retained in equations (13) and (14) for flow at a distance from the wall, curves in figures 3 and 4 were plotted using  $y_1^+$  equal to 16 as was used in reference 1 for air when the molecular-shear-stress and heat-transfer terms were retained. The peculiar variation of the physical properties of supercritical water necessitates the calculation of velocity and temperature distributions for each wall temperature. In general, the variation of the velocity distributions is more regular than that of the temperature distributions. The irregular variation of the temperature distribution is due to the unusual variation of specific heat with temperature. In equation (18) the specific heat appears in the denominator.

### Nusselt Numbers

In figures 5 and 6, the predicted Nusselt numbers for supercritical water are plotted against Reynolds numbers for various values of the heat-transfer parameter  $\beta$  for a pressure of 5000 pounds per square inch and a wall temperature of  $1360^{\circ}$  R.

Figure 5 shows the Nusselt numbers plotted against the Reynolds numbers with all the physical properties evaluated at the wall temperature. The Nusselt numbers and Reynolds numbers are calculated from equations (20) and (21). At a given Reynolds number, the Nusselt numbers can be seen to increase in a complex manner as  $\beta$  increases. In figure 6, all the physical properties in the Reynolds numbers and Nusselt numbers are evaluated at the bulk temperature. For a given Reynolds number, the Nusselt numbers decrease irregularly with an increase in  $\beta$ . Because of the irregular variation of the Nusselt number with  $\beta$ , no single reference temperature  $t_x$  can be found for the evaluation of the physical properties which will eliminate the effects of  $\beta$ . The effects of  $\beta$  can be eliminated, however, by using a reference temperature which is a function of both the wall temperature and the ratio of wall-to-bulk temperature. In figure 7 the Nusselt numbers are plotted against the Reynolds numbers with the physical properties, including density, evaluated at the reference temperature for a pressure of 5000 pounds per square inch and various wall temperatures. These are the curves for  $\beta = 0$  for the various wall temperatures and can be used for  $\beta \neq 0$  by using the reference temperatures from figure 8, where  $x$  is plotted against the ratio of wall-to-bulk temperature for the various wall temperatures. The quantity  $x$ , which defines the reference temperature  $t_x = x(t_0 - t_b) + t_b$ , is calculated from the curves in figure 5 and the corresponding curves for other wall temperatures. A reference temperature can be selected for each point on the curves for  $\beta \neq 0$  which brings the point to the  $\beta = 0$  curve. The quantity  $x$  varies widely with both wall temperature and ratio of wall-to-bulk temperature.

The variation of the level of the curves for Nusselt numbers plotted against Reynolds number in figure 7 is caused by the variation in Prandtl number. These curves indicate that Nusselt number varies approximately as  $Pr_0^{0.55}$  for the range of Prandtl numbers shown, whereas experimental data for various fluids indicate a variation of  $Pr_0^{0.45}$ <sup>1</sup>. This difference is probably a result of neglecting the effect

<sup>1</sup>An examination of the data for Prandtl numbers in the range considered in this report, for example, in references 7 and 8, indicates that an exponent of 0.45 fits the data better than the commonly used exponent 0.4, which is an average for a larger range of Prandtl numbers than considered herein.

of kinematic viscosity of the eddy diffusivity in the region very close to the wall; this region becomes increasingly important as the Prandtl number is increased. If a more accurate answer is desired, the present recommended procedure is to calculate  $Nu_x$  by multiplying the  $Nu_x$ 's for a Prandtl number of 1 in figure 7 by the correction factor,  $Pr_0^{0.45}$  instead of using the curves for various wall temperatures in figure 7. The reference temperature for evaluating the properties in the Nusselt and Reynolds numbers can be obtained from figure 8.

As an application of the foregoing analysis, wall and bulk temperatures for flow through a 0.25-inch tube with uniform heat input were calculated and are shown in figure 9. The bulk temperatures were obtained by setting the total heat transferred between the entrance and a given section equal to the change in enthalpy of the fluid passing the section and determining the temperature change corresponding to the enthalpy change (constant pressure) from the tables in reference 2. The curves for wall and bulk temperatures are nearly parallel at the lower temperatures. As the bulk temperature approaches the critical temperature, the curves separate because of the rapid variation of physical properties in that region. The changes in slope of the bulk-temperature curve are caused by the very large increase in specific heat with temperature in the critical region and the subsequent decrease at higher temperatures. The wall-temperature curve does not follow the bulk-temperature curve because the properties of the fluid change from those of a liquid to those of a gas in passing through the critical region, with a consequent decrease in heat-transfer coefficient. It should be mentioned that the increase in specific heat near the critical temperature tends to increase the heat-transfer coefficient because of the larger Prandtl number in that region; however, that effect is more than offset by the reduction in the values of the other properties.

Several analyses which utilize assumptions other than those made in the present analysis were made and are reported in reference 3. Therein Goldman assumed that the curve of a plot of  $u^+$  against  $y^+$  for  $\beta = 0$  can

be used for flow with heat transfer if  $u^+$  is redefined as  $\int_0^u \frac{du}{\sqrt{\nu_0/\rho}}$

and  $y^+$  is written as  $\int_0^y \frac{\sqrt{\nu_0/\rho}}{\mu/\rho} dy$ , where local values of the prop-

erties are used. Also, Elrod utilized the properties of Prandtl's mixing-length theory and a laminar layer at the wall. The results of these two analyses are in reasonable agreement, as shown in reference 3. A comparison between these analyses and the present one is made in figure 10. Most of the difference in calculated heat flux is caused by the difference in the viscosities and thermal conductivities used (fig. 2). The

curves in figure 10 calculated from the present analysis and the properties from reference 3 are in reasonable agreement with the analyses from reference 3. This agreement indicates that the calculated heat transfer is insensitive to many of the assumptions made in the analysis.

### Friction Factor

The predicted friction factors plotted against Reynolds numbers for various values of the heat-transfer parameter  $\beta$  for a pressure of 5000 pounds per square inch and a wall temperature of  $1360^{\circ}$  R are shown in figures 11 and 12.

In figure 11, all the physical properties in the friction factors and Reynolds numbers are evaluated at the wall temperature. For a given Reynolds number, an increase in the friction factors occurs with an increase in  $\beta$ . The friction factors plotted against Reynolds numbers with the physical properties evaluated at the bulk temperature are shown in figure 12. At a given Reynolds number, the friction factors can be seen to decrease nonuniformly with an increase in  $\beta$ . Since the friction factors vary in a complex manner with  $\beta$ , no single reference temperature  $t_x$  can be found for the evaluation of the physical properties which will eliminate the effects of  $\beta$ . To eliminate the effects of  $\beta$ , a reference temperature which is a function of both the wall temperature and the ratio of wall-to-bulk temperature must be used. With all the physical properties evaluated at this reference temperature, the friction factors are plotted against the Reynolds numbers in figure 13. A plot of  $x$  against the ratio of wall-to-bulk temperature for the various wall temperatures, where  $x$  is defined by  $t_x = x(t_0 - t_b) + t_b$ , is shown in figure 14. The values of  $x$  were computed from the curves in figure 12 and the corresponding curves for other wall temperatures. There is a variation of  $x$  with wall temperature and ratio of wall-to-bulk temperature for the friction factors, but it is not so great as the variation for the Nusselt numbers.

### SUMMARY OF RESULTS

The following results were obtained from the analytical investigation of turbulent flow and heat transfer of supercritical water in smooth tubes:

1. Heat addition to the fluid caused a flattening of the velocity and temperature profiles. The variation of temperature distribution with heat addition was complex because of the unusual variation of specific heat with temperature.

2. The effects of variable properties on the Nusselt number and friction factor correlations were eliminated by evaluating the properties of the Nusselt number, Reynolds number, and friction factor at reference temperatures which are functions of both wall temperature and ratio of wall-to-bulk temperature.

Lewis Flight Propulsion Laboratory  
National Advisory Committee for Aeronautics  
Cleveland, Ohio, January 30, 1953

## APPENDIX - SYMBOLS

The following symbols are used in this report:

$c_p$	specific heat of fluid at constant pressure, Btu/(lb)(°F)
$c_{p,0}$	specific heat of fluid at constant pressure evaluated at $t_0$ , Btu/(lb)(°F)
D	inside diameter of tube, ft
e	base of natural logarithms
g	acceleration due to gravity, 32.2 ft/sec <sup>2</sup>
h	heat-transfer coefficient, $\frac{q_0}{t_0 - t_b}$ , Btu/(sec)(sq ft)(°F)
k	thermal conductivity of fluid, Btu/(sec)(sq ft)(°F/ft)
$k_b$	thermal conductivity of fluid evaluated at $t_b$ , Btu/(sec)(sq ft)(°F/ft)
$k_0$	thermal conductivity of fluid evaluated at $t_0$ , Btu/(sec)(sq ft)(°F/ft)
$k_x$	thermal conductivity of fluid evaluated at $t_x$ , Btu/(sec)(sq ft)(°F/ft)
n	constant
p	static pressure, lb/sq ft abs
q	rate of heat transfer toward tube center per unit area, Btu/(sec)(sq ft)
$q_0$	rate of heat transfer at inside wall toward tube center per unit area, Btu/(sec)(sq ft)
$r_0$	inside tube radius, ft
t	absolute static temperature, °R
$t_0$	absolute wall temperature, °R
$t_b$	bulk or average static temperature of fluid at cross section of tube, °R
$t_x$	film temperature, $x(t_0 - t_b) + t_b$ , °R

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- u time-average velocity parallel to axis of tube, ft/sec
- $u_b$  bulk or average velocity at cross section of tube, ft/sec
- x number used in evaluating arbitrary temperature in tube,  $t_x$ , ft
- y distance from tube wall, ft
- $\epsilon$  coefficient of eddy diffusivity for momentum, sq ft/sec
- $\epsilon_h$  coefficient of eddy diffusivity for heat, sq ft/sec
- $\kappa$  Kármán constant
- $\mu$  absolute viscosity of fluid, (lb)(sec)/sq ft
- $\mu_b$  absolute viscosity of fluid evaluated at  $t_b$ , (lb)(sec)/sq ft
- $\mu_0$  absolute viscosity of fluid evaluated at  $t_0$ , (lb)(sec)/sq ft
- $\mu_x$  absolute viscosity of fluid evaluated at  $t_x$ , (lb)(sec)/sq ft
- $\rho$  mass density, (lb)(sec<sup>2</sup>)/ft<sup>4</sup>
- $\rho_b$  bulk or average density at cross section of tube, (lb)(sec<sup>2</sup>)/ft<sup>4</sup>
- $\rho_0$  mass density of fluid at wall, (lb)(sec<sup>2</sup>)/ft<sup>4</sup>
- $\rho_x$  density of fluid evaluated at  $t_x$ , (lb)(sec<sup>2</sup>)/ft<sup>4</sup>
- $\tau$  shear stress in fluid, lb/sq ft
- $\tau_0$  shear stress in fluid at wall, lb/sq ft

#### Subscripts:

- fr on friction-pressure gradient

#### Dimensionless groups:

$$\beta \text{ heat-transfer parameter, } \frac{q_0 \sqrt{\tau_0 / \rho_0}}{c_{p,0} g t_0 t_0}$$

$$f_0 \text{ friction factor with density evaluated at } t_0, \frac{D(dp/dx)_{fr}}{2} = \frac{2\tau_0}{\rho_0 u_b^2}$$

$f_b$	friction factor with density evaluated at $t_b$ , $\frac{2\tau_0}{\rho_b u_b^2}$
$f_x$	friction factor with density evaluated at $t_x$ , $\frac{2\tau_0}{\rho_x u_b^2}$
$Nu_0$	Nusselt number with thermal conductivity evaluated at $t_0$ , $\frac{hD}{k_0}$
$Nu_b$	Nusselt number with thermal conductivity evaluated at $t_b$ , $\frac{hD}{k_b}$
$Nu_x$	Nusselt number with thermal conductivity evaluated at $t_x$ , $\frac{hD}{k_x}$
$Pe$	Peclet number
$Pr$	Prandtl number
$Pr_0$	Prandtl number with properties evaluated at wall, $c_{p,0} \mu_0 g / k_0$
$Re$	Reynolds number
$Re_0$	Reynolds number with density and viscosity evaluated at $t_0$ , $\frac{\rho_0 u_b D}{\mu_0}$
$Re_b$	Reynolds number with density and viscosity evaluated at $t_b$ , $\frac{\rho_b u_b D}{\mu_b}$
$Re_x$	Reynolds number with density and viscosity evaluated at $t_x$ , $\frac{\rho_x u_b D}{\mu_x}$
$r_0^+$	tube-radius parameter, $\frac{\sqrt{\tau_0/\rho_0}}{\mu_0/\rho_0} r_0$
$t^+$	temperature parameter, $\frac{(t_0-t) c_{p,0} g \tau_0}{a_0 \sqrt{\tau_0/\rho_0}} \frac{1-t/t_0}{\beta}$
$t_1^+$	value of $t^+$ at $y_1^+$
$t_b^+$	bulk-temperature parameter, $\frac{1}{\beta} \left(1 - \frac{t_b}{t_0}\right)$
$u^+$	velocity parameter, $\frac{u}{\sqrt{\tau_0/\rho_0}}$

$u_b^+$  bulk velocity parameter,  $\frac{u_b}{\sqrt{\tau_0/\rho_0}}$

$u_1^+$  value of  $u^+$  at  $y_1^+$

$v$   $\frac{du^+}{dy^+}$

$v_1$  value of  $v$  at  $y_1^+$

$y^+$  wall-distance parameter,  $\frac{\sqrt{\tau_0/\rho_0}}{\mu_0/\rho_0} y$

$y_1^+$  value of  $y^+$  at intersection of curves for flow close to wall and at a distance from wall

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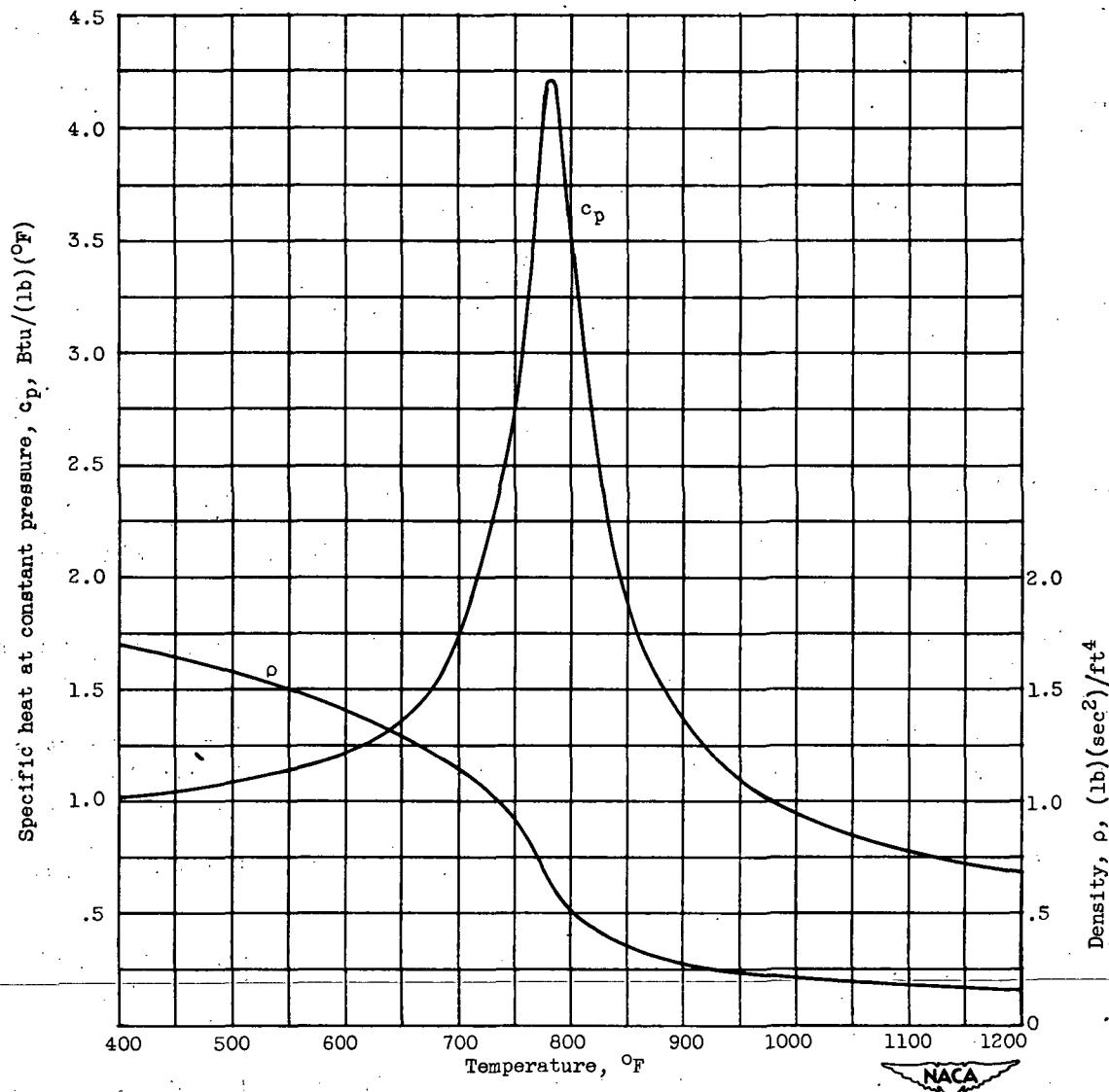


Figure 1. - Variation of density and specific heat of water with temperature.  
Pressure, 5000 pounds per square inch. Data obtained from references 2  
and 4.

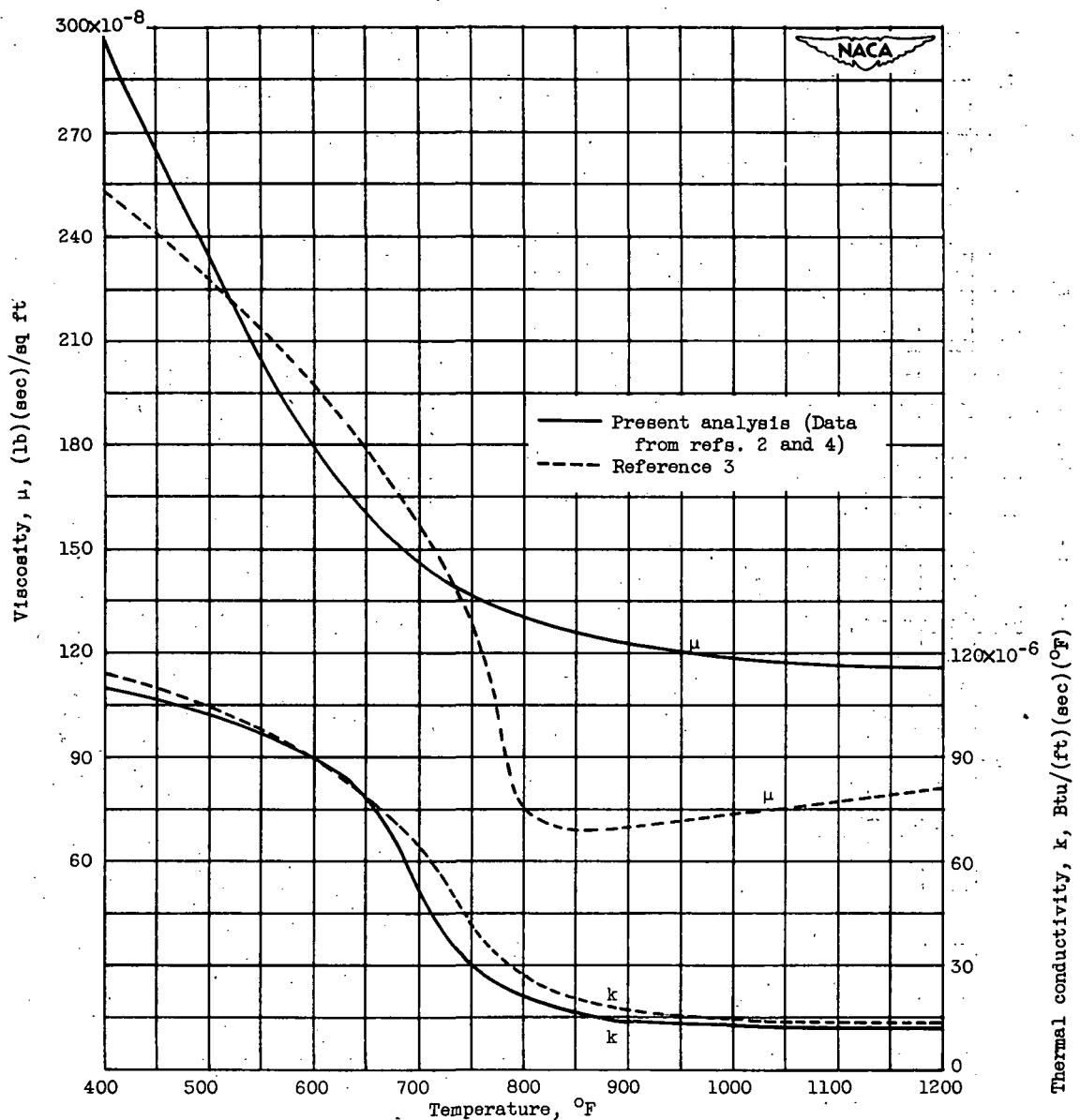


Figure 2. - Variation of thermal conductivity and viscosity of water with temperature. Pressure, 5000 pounds per square inch.

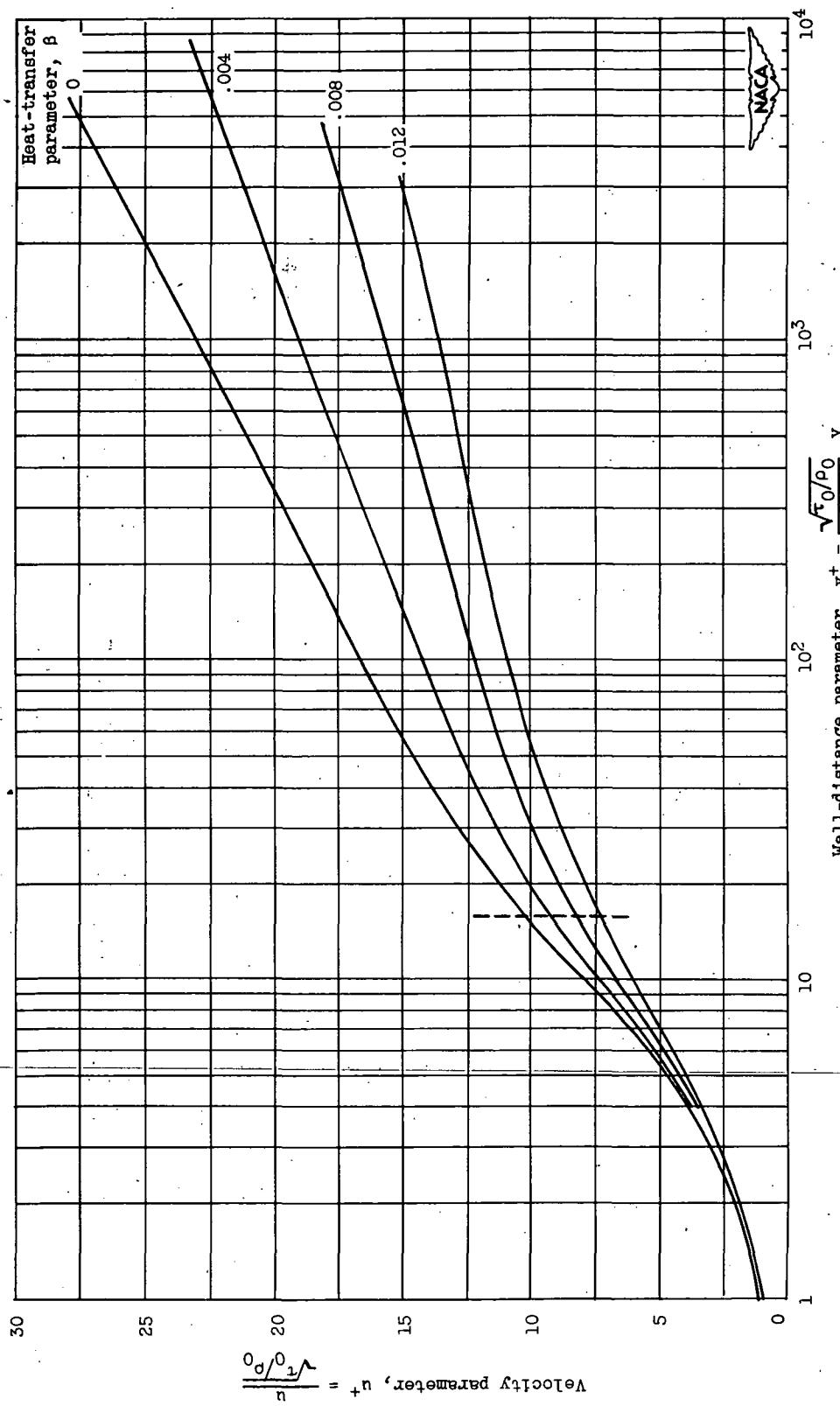


Figure 3. - Predicted generalized velocity distributions for flow of supercritical water with heat addition. Pressure, 5000 pounds per square inch; wall temperature, 1360° R.

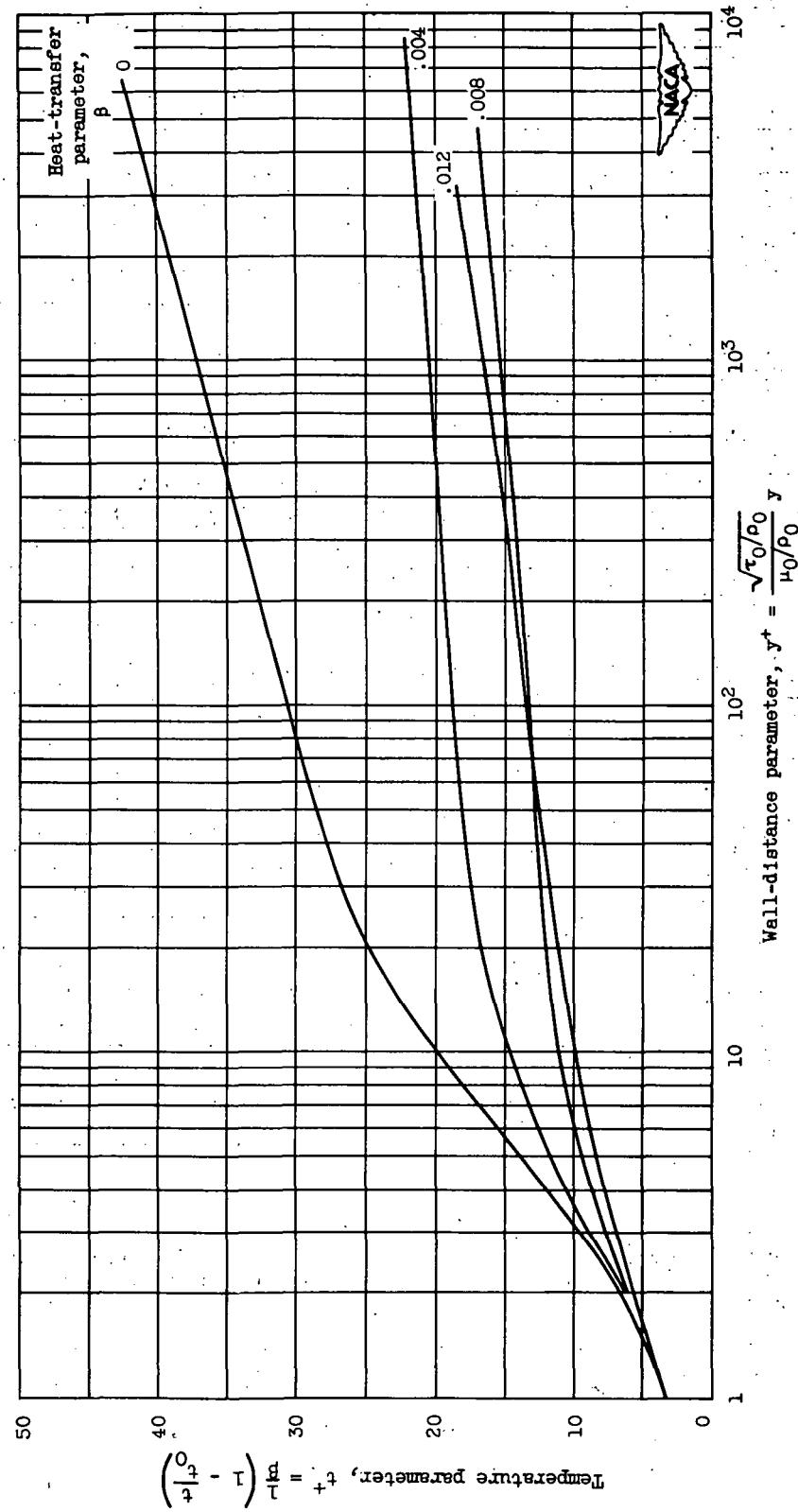


Figure 4. - Predicted generalized temperature distributions for flow of supercritical water with heat addition. Pressure, 5000 pounds per square inch; wall temperature, 1360° R.

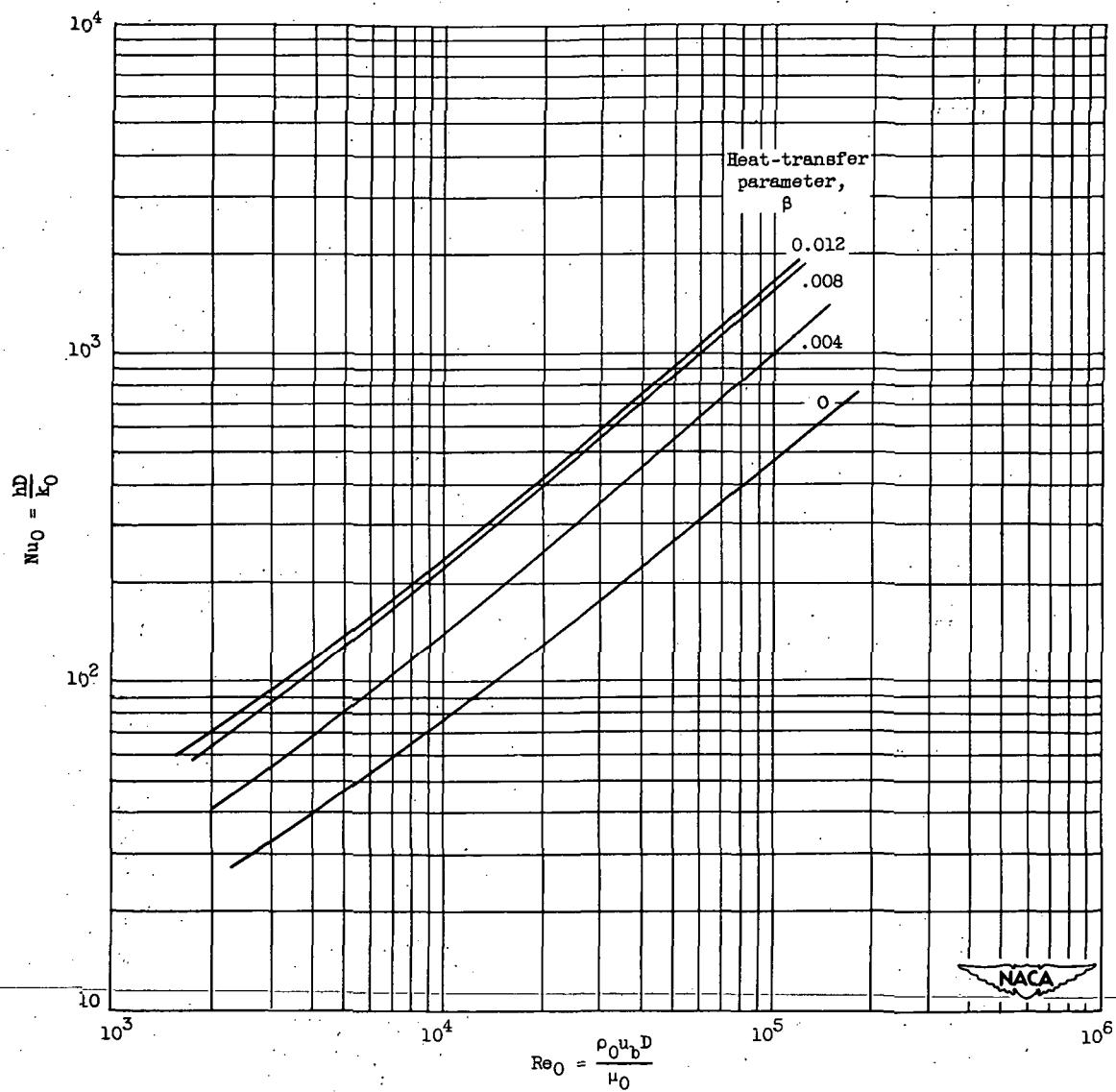


Figure 5. - Predicted variation of Nusselt number with Reynolds number for flow of supercritical water with heat addition and properties evaluated at wall temperature. Pressure, 5000 pounds per square inch; wall temperature,  $1360^{\circ}$  R.

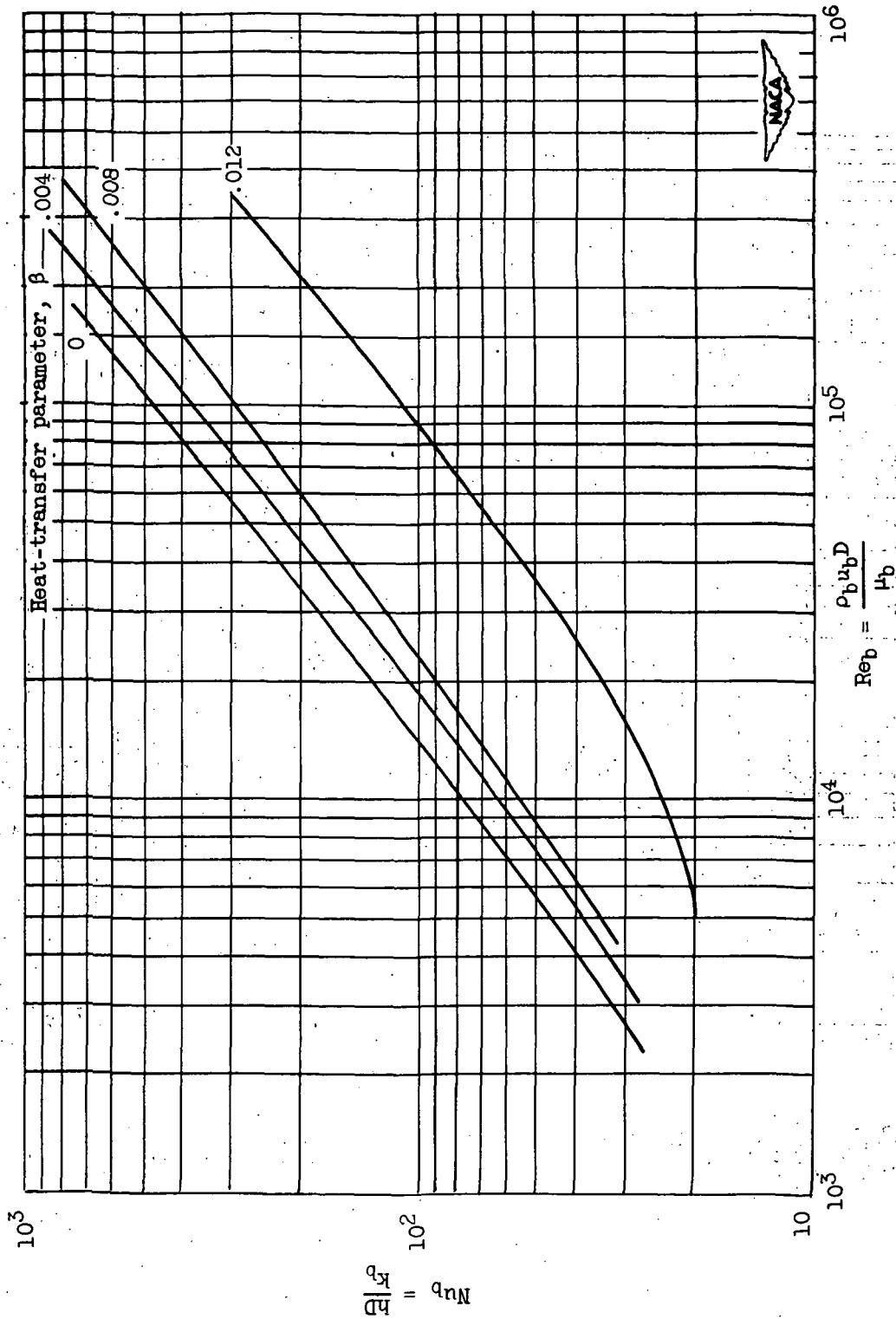


Figure 6. - Predicted variation of Nusselt number with Reynolds number for flow of supercritical water with heat addition and properties evaluated at bulk temperature. Pressure, 5000 pounds per square inch; wall temperature,  $1360^{\circ}$  R.

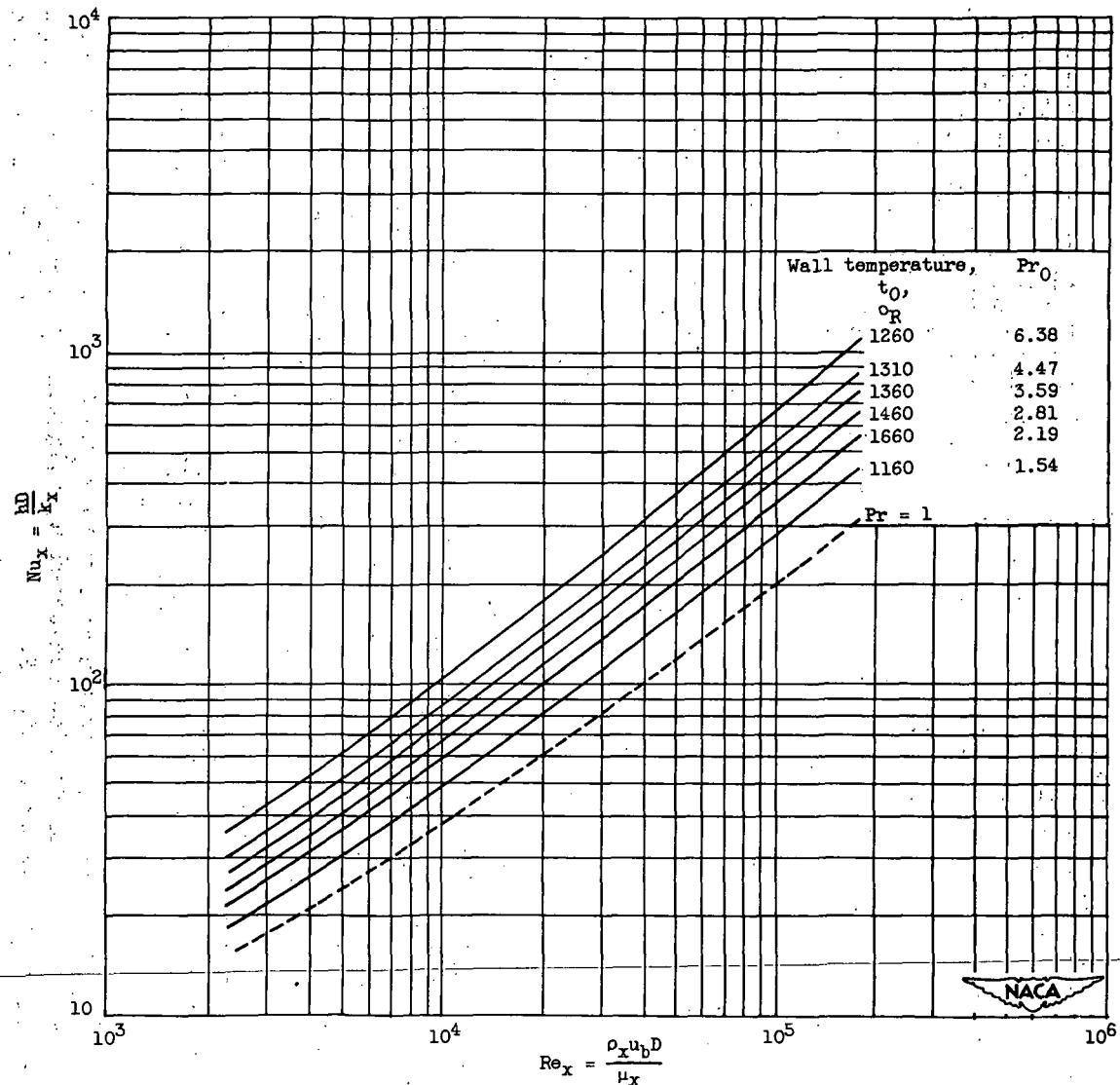


Figure 7. - Predicted variation of Nusselt number with Reynolds number for flow of supercritical water with heat addition and properties evaluated at reference temperature. Pressure, 5000 pounds per square inch.

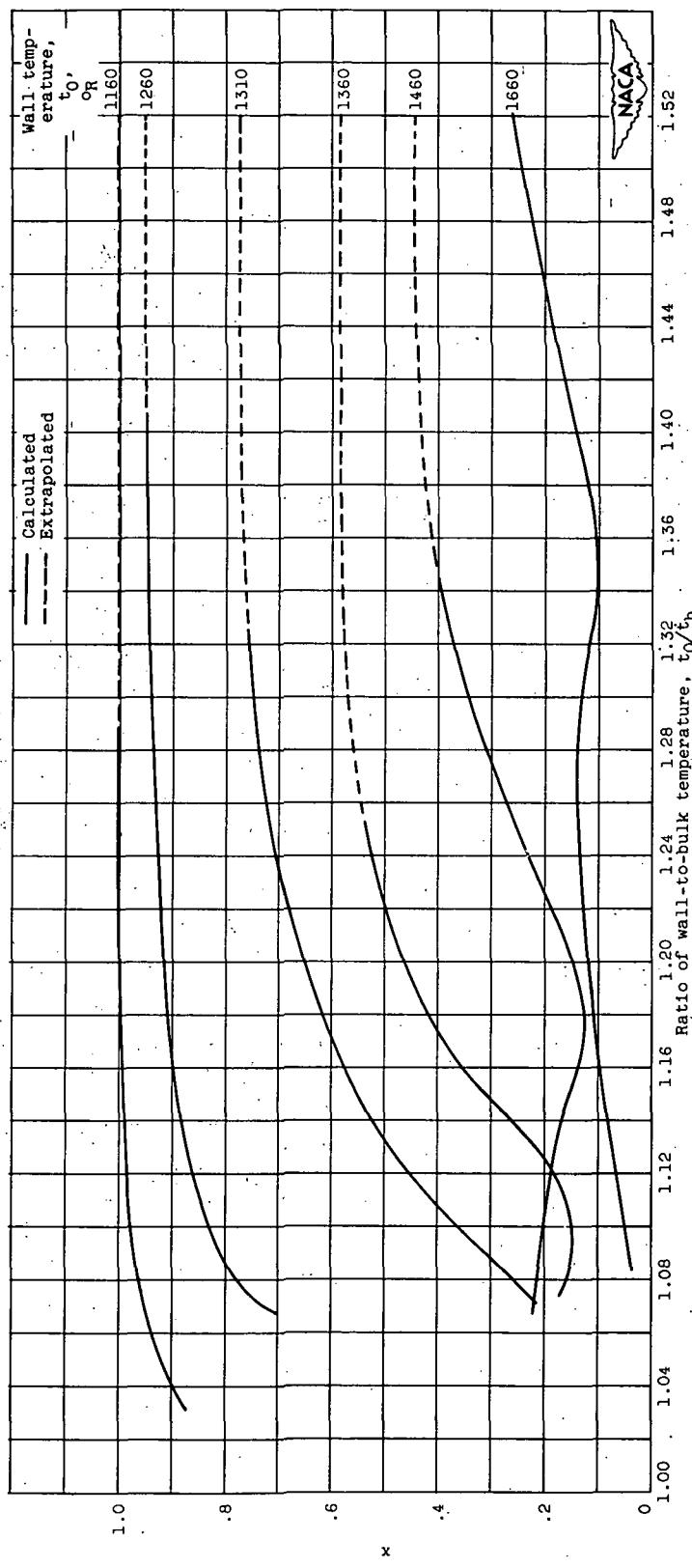


Figure 8. - Variation of  $x$  as defined by  $t_x = x(t_0 - t_b) + t_b$  with wall temperature and ratio of wall-to-bulk temperature for determination of reference temperatures for use with Nusselt number curves in figure 7. Pressure, 5000 pounds per square inch.

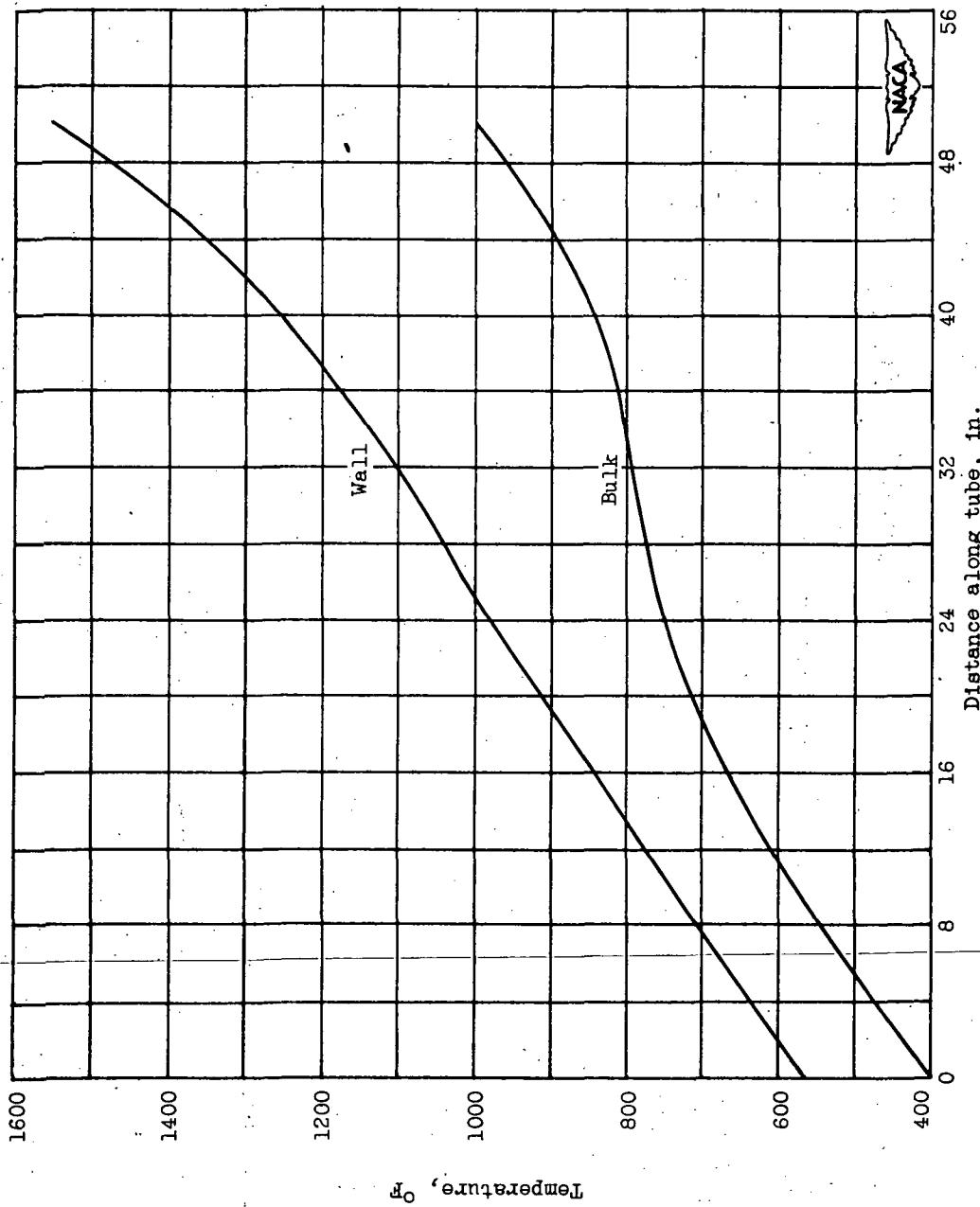


Figure 9. - Predicted variation of wall and bulk temperatures for flow through 0.25-inch tube with constant heat input using curves in figure 7. Heat input, 575 Btu per second per square foot; weight flow, 0.0198 pound per second; pressure, 5000 pounds per square inch.

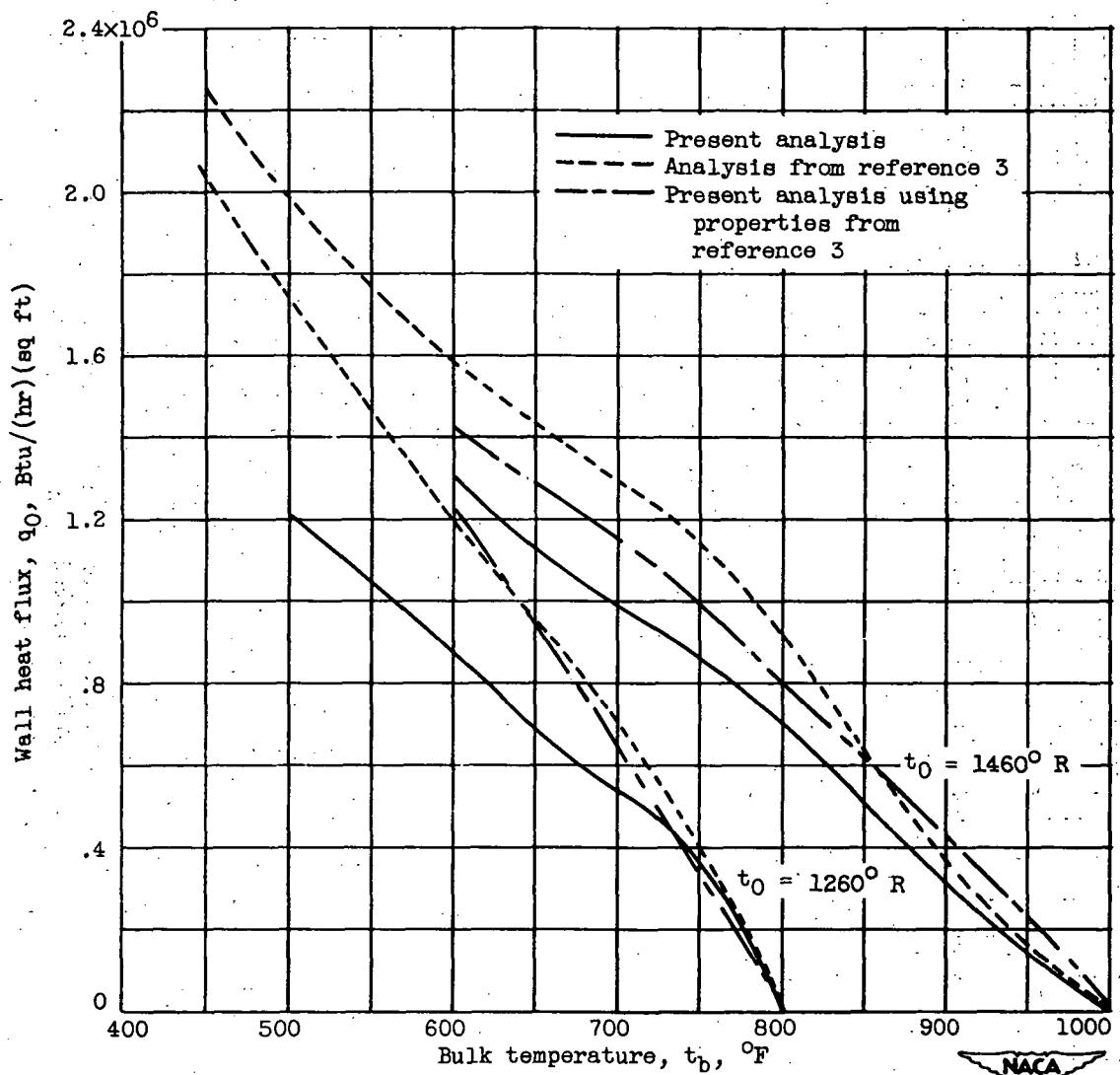


Figure 10. - Comparison of heat flux predicted by reference 3 and in present analysis for wall temperatures of  $1260^{\circ}$  and  $1460^{\circ}$  R. Tube diameter, 0.00521 foot; weight flow, 0.0118 pound per second; pressure, 5000 pounds per square inch.



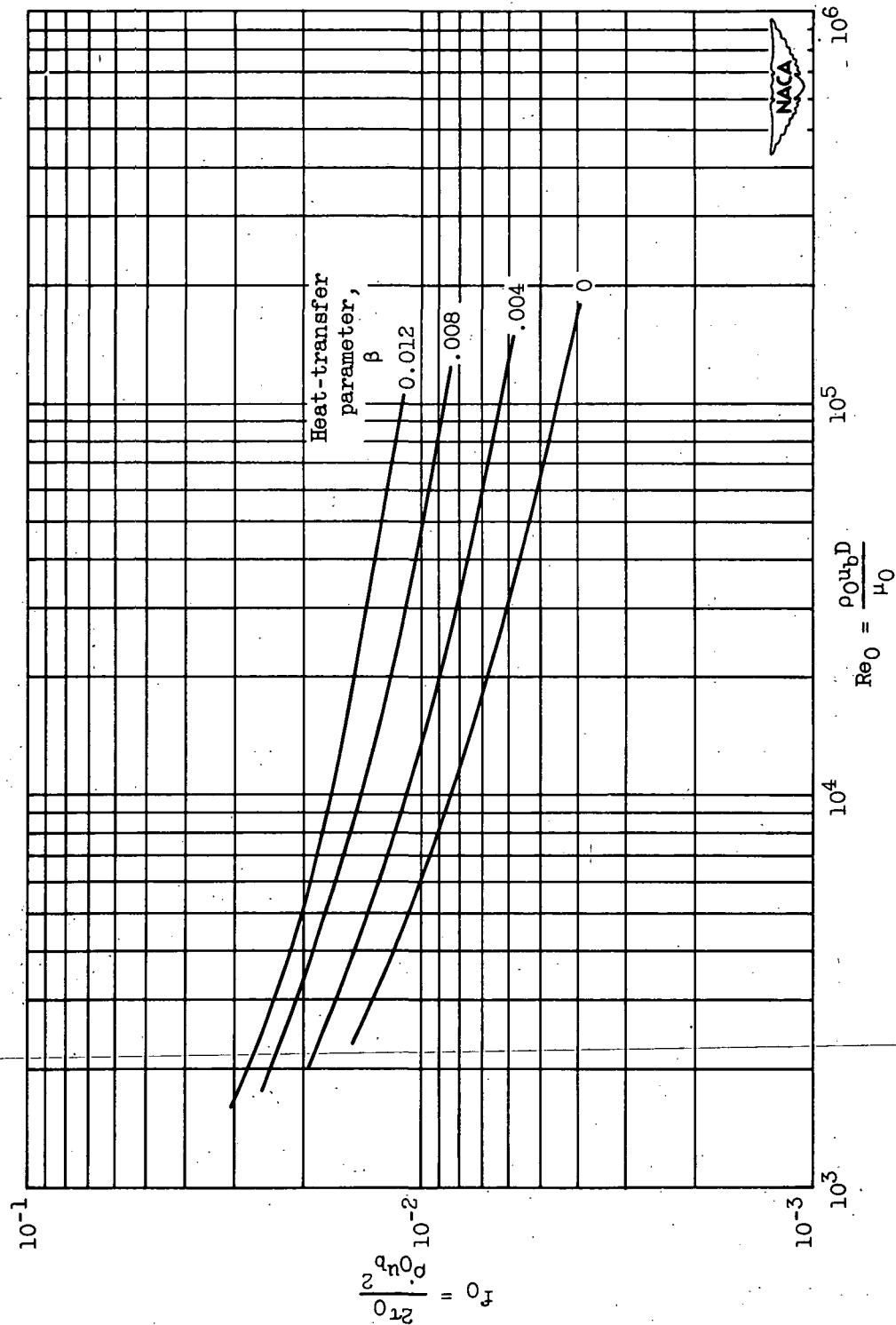


Figure 11. - Predicted variation of friction factor with Reynolds number for flow of supercritical water with heat addition and properties evaluated at wall temperature. Pressure, 5000 pounds per square inch; wall temperature, 1360° R.

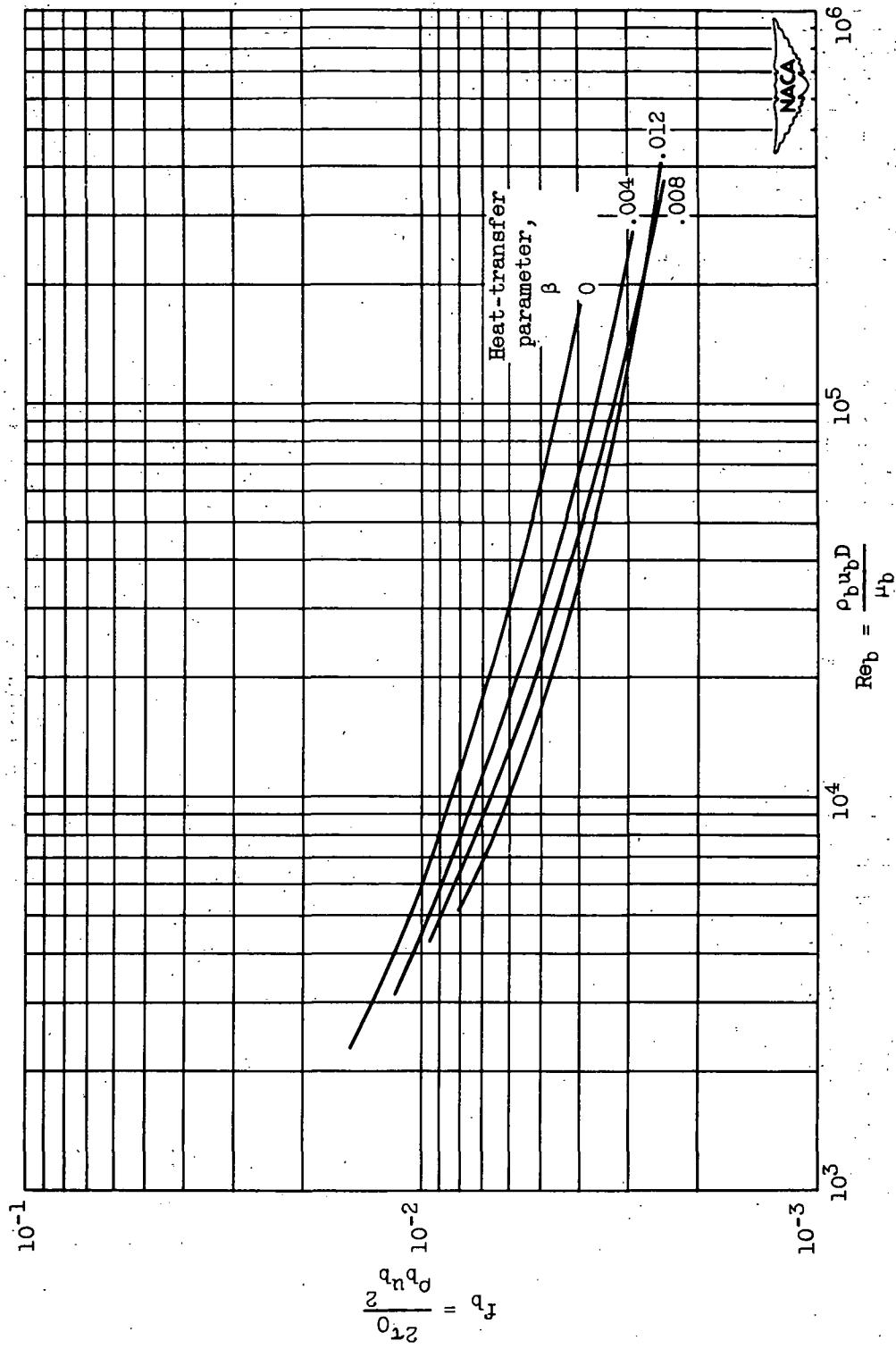


Figure 12. - Predicted variation of friction factor with Reynolds number for flow of supercritical water with heat addition and properties evaluated at bulk temperature. Pressure, 5000 pounds per square inch; wall temperature,  $1360^{\circ}$  R.

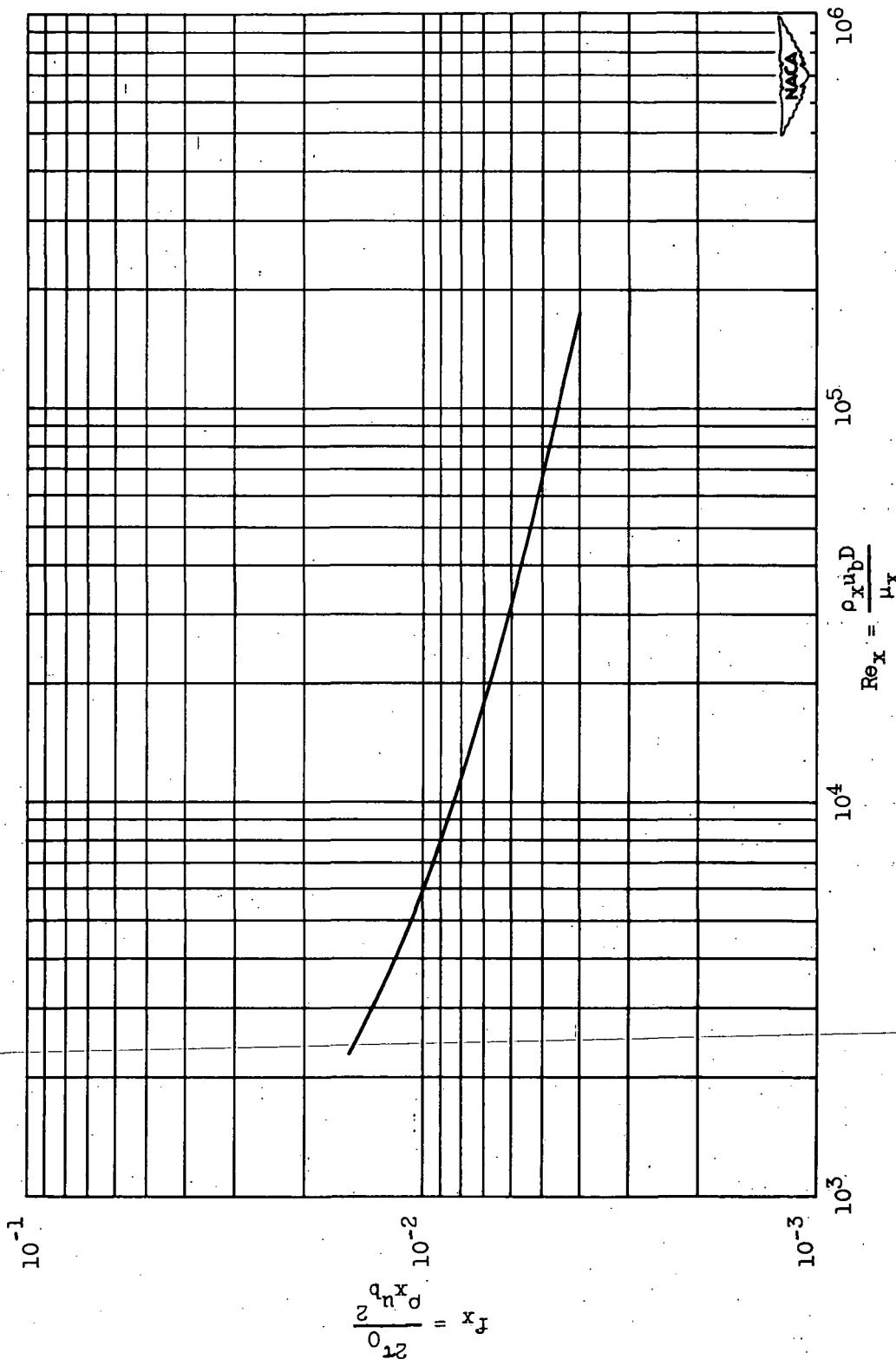


Figure 13. - Predicted variation of friction factor with Reynolds number for flow of supercritical water with heat addition and properties evaluated at reference temperature.

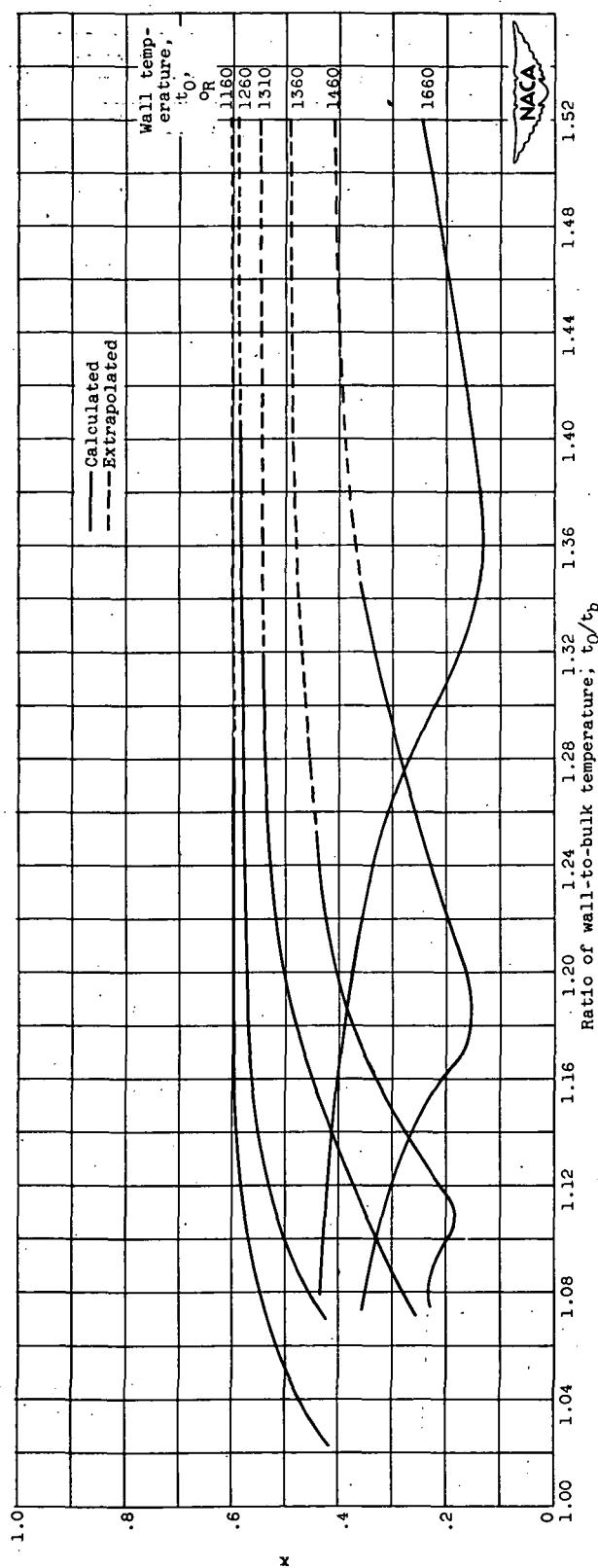


Figure 14. - Variation of  $x$ , as defined by  $t_x = x(t_0 - t_b) + t_b$ , with wall temperature and ratio of wall-to-bulk temperature for determination of reference temperatures for use with friction factor curves in figure 13. Pressure, 5000 pounds per square inch.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

ANALYSIS OF HEAT TRANSFER AND FLUID FRICTION FOR FULLY  
DEVELOPED TURBULENT FLOW OF SUPERCRITICAL WATER WITH  
VARIABLE FLUID PROPERTIES IN A SMOOTH TUBE

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Flow, Turbulent	1.1.3.2
Pipes	1.4.2.3
Heat-Transfer Theory and Experiment	3.9.1
Deissler, Robert G., and Taylor, Maynard	

Abstract

A previous analysis of turbulent flow and heat transfer for air with variable properties flowing in smooth tubes is generalized in order to make it applicable to supercritical water. The generalization is necessary because all the pertinent properties of supercritical water vary markedly with temperature. Calculated velocity and temperature distributions, as well as relations among Nusselt number, Reynolds number, and friction factor, are presented. The effect of variation of fluid properties across the tube on the Nusselt number and friction factor correlations can be eliminated by evaluating the properties at a reference temperature which is a function of both the wall temperature and the ratio of wall-to-bulk temperatures.